Network diversification for a robust portfolio allocation

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FinanceHub, 20th/Jun/2022

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Introduction

Financial futures span different markets, asset classes and geographical areas. Investors desire strategies that allocate assets in ways that perform in all market regimes.

Markowitz portfolio – a lot of instability in the estimation. Weights are sensitive on the input parameters, specially the expected returns. It usually produces a concentrated portfolio.

Minimum variance portfolio (MVP) – independent of expected returns. $\arg\min_{w} \frac{1}{2} w^T \Sigma w$

"Heuristics" portfolio (Roncalli (2013)):

- **Equal Weight** (1/N)
- **Risk Parity** (equal risk)

Risk Parity

Different methods to create Risk Parity portfolios:

- Inverse volatility – Naïve Risk Parity (NRP) $w_i = \frac{\sigma_i^{-1}}{\sum_j \sigma_j^{-1}}.$

- Equal Risk Contribution (ERC) $RC_i = \frac{w_i(\Sigma w)_i}{\sqrt{w'\Sigma w}}$ $\arg\min_w \sum_i \left(\frac{RC_i}{\sqrt{w'\Sigma w}} - \frac{1}{N} \right)$

- Most Diversified Portfolio (MDP) $DR(w) = \frac{\sum_i w_i \sigma_i}{\sigma(w)}$ $\arg\max_w DR(w)$

- Hierarchical Risk Parity (HRP)
Cluster according to some distance metric and then allocate equal risk budget along these clusters. Lopez de Prado (2016)

Drawback:
- Correlation matrices need to be estimated. Dependence is dynamical. Large samples are needed, but older data points are less representative of the current environment.

Networks

A network is a graph with vertices V = {1, …, N} and edges E = {1, …, M}. It express a simplified representation of the interaction between entities.

Assets = vertices and dependence = edges.
Whole correlation matrix: N x (N-1) / 2 edges.
Filter only relevant connections.

Weights of the edges: distance measure (inverse of dependence)

Mantegna (1999): use minimum spanning tree (MST), a kind of network, to Financial Theory.

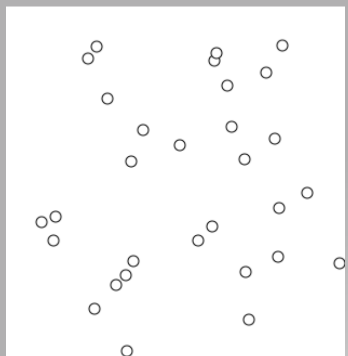
Onnela (2003): prove that the assets of the Markowitz portfolio are always located on the outer leaves of the MST.

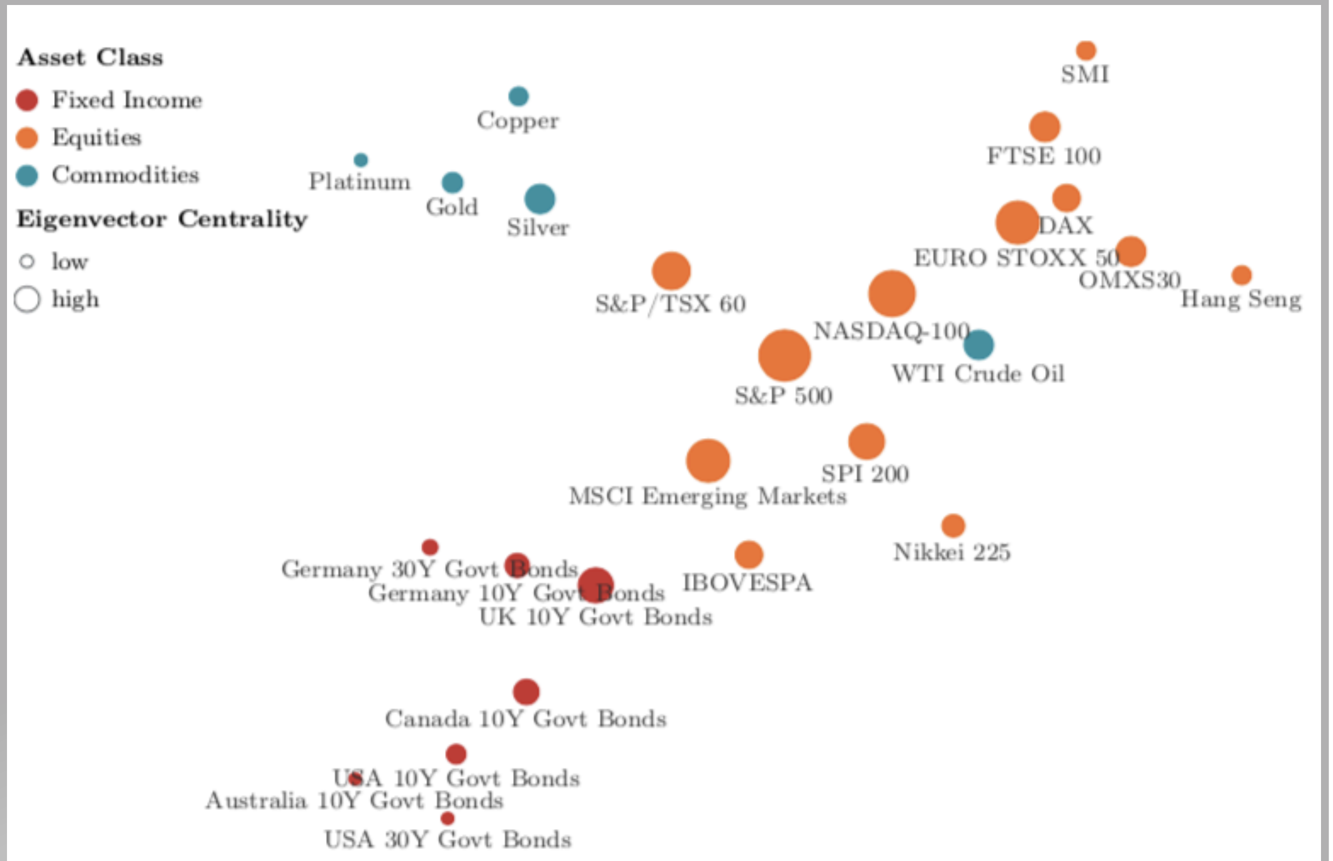
Minimum Spanning Tree

Connect all vertices minimizing distance.

N – 1 edges

Kruskal (1956)





Planar Maximally Filtered Graph (PMFG)

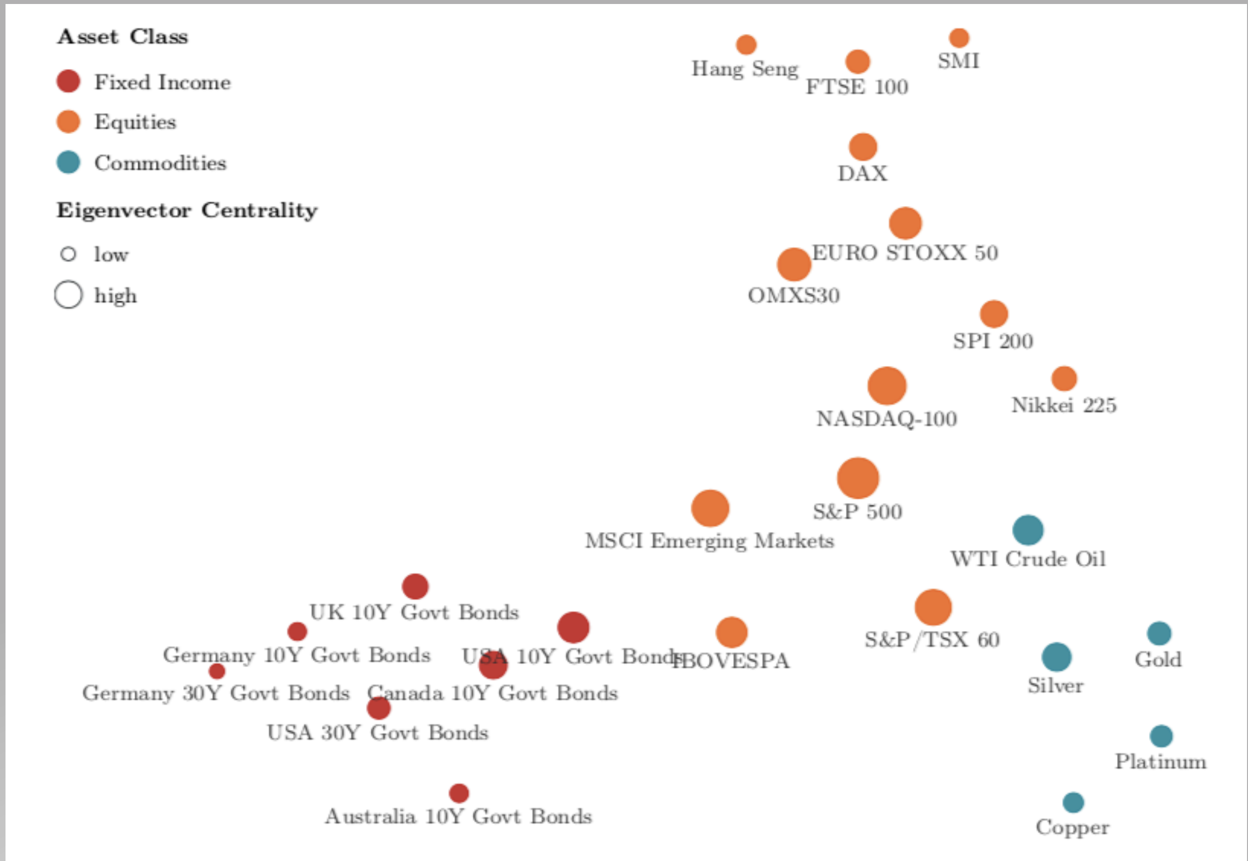
Tumminello et al (2005)

Planar = non-overlapping edges

Maximum # of edges to remain planar

3N – 6 edges

Boyer and Myrvold (2004)



Multiplex Network

Multiplex network: each layer has the same vertices (markets), but different levels on information on the edges.

Musmeci et al (2017) – 4-layer network (Pearson, Kendall, Tail and Partial) correlations.

In this work, a 3-layer multiplex network is used:
- Pearson correlation
- Kendall correlation
- Lower tail dependence

$$d^{\rho}_{i,j} = \sqrt{\frac{1}{2}(1-\rho_{i,j})}$$

$$d^{\tau}_{i,j} = \sqrt{\frac{1}{2}(1-\kappa_{i,j})}$$

$$\lambda_l = \lim_{t \downarrow 0} \mathbb{P}(X \leq F_X^{-1}(t) | Y \leq F_Y^{-1}(t))$$

Network measures

- Degree centrality: number of incoming edges to each vertex

$D_{MST} = 1 – 1/N$

$D_{PMFG} = 3 – 6/N$

- Eigenvector centrality:

A: adjacency matrix

aij = 1 if vertex i is connected to vertex j

= 0 otherwise

Find largest eigenvalue λ such that, for x:

$A\ x = \lambda\ x$

Normalize x to sum one.

x is the eigenvector centrality measure

Multilayer: Sum all the A matrices and find eigenvector

Network-based allocation

- Pozzi, Di Matteo and Aste (2013): show that selecting peripheral assets of the network can improve the risk-return of a portfolio.

- Peraltaa and Zareei (2016): negative relationship between centrality of assets and their optimal weights in Markowitz.

- Vyrost, Lyocsa and Baumohl (2019) use MST and PMFG to build multi-asset portfolios.

-Degree centrality: $$w_i^{IDCP} = \frac{(\sigma_i DC_i)^{-1}}{\sum_j (\sigma_j DC_j)^{-1}}$$

- Eigenvector centrality: $$w_i^{IECP} = \frac{(\sigma_i EC_i)^{-1}}{\sum_j (\sigma_j EC_j)^{-1}}$$

Models considered

- IDCP$^{\rho}$: the inverse degree centrality portfolio applied to the planar maximally filtered graph (PMFG) based on Pearson correlation.
- IECP$^{\rho}$: the inverse eigenvector centrality portfolio applied to the PMFG based on the Pearson correlation.
- IECP^{l}: the inverse eigenvector centrality portfolio applied to the PMFG based on the lower tail dependence.
- IECP$^{\rho,\tau,l}$: the inverse eigenvector centrality portfolio applied to the multiplex network with three PMFG-layers defined by the Pearson correlation, Kendall's tau and the lower tail dependence. The eigenvector centrality of the multiplex network is determined by the uniform eigenvector-like centrality.

Dataset

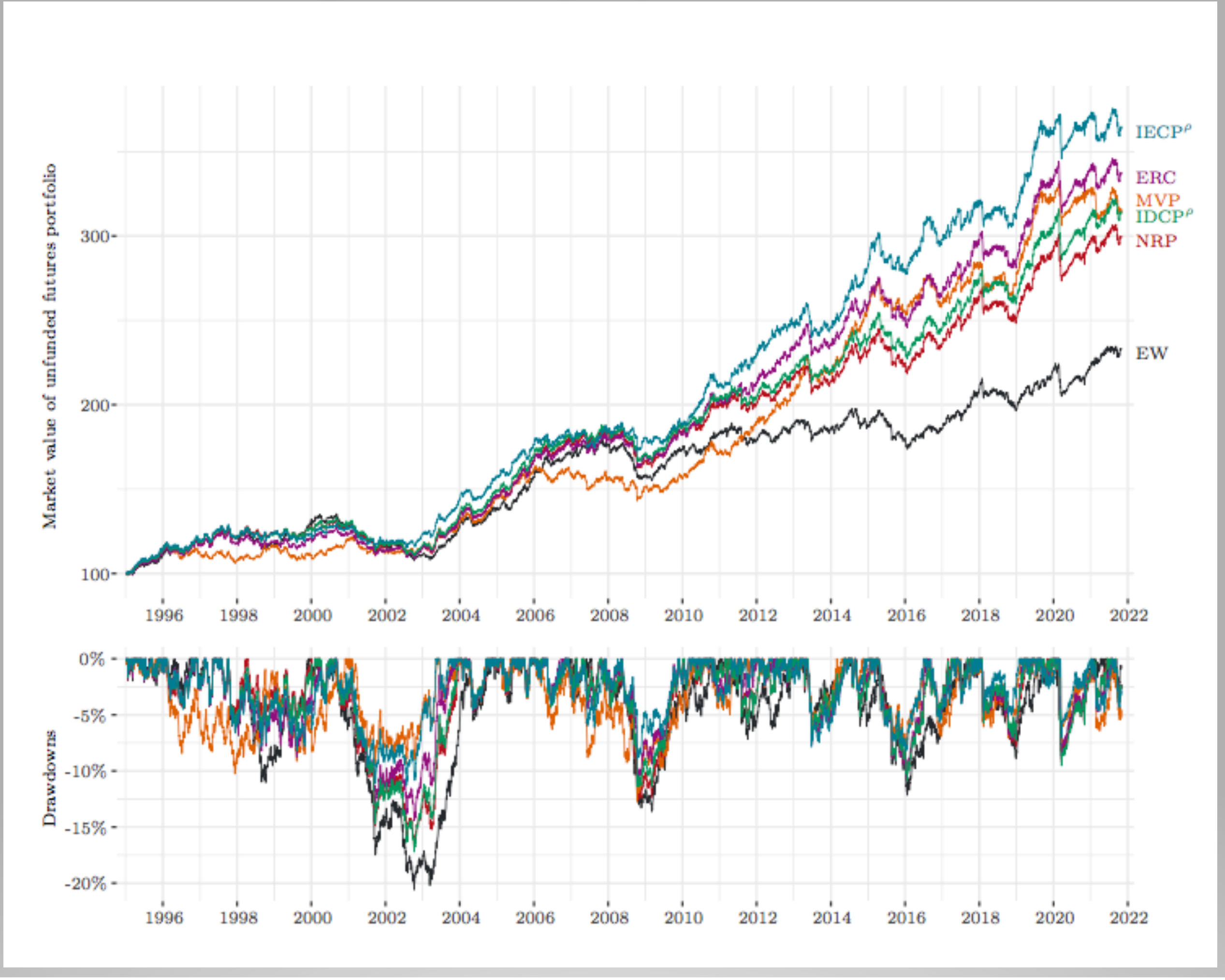
ID	Constituent	Asset Class	Currency	ID	Constituent	Asset Class	Currency
1	Copper	Commodities	USD	14	OMXS30	Equities	SEK
2	Gold	Commodities	USD	15	S&P 500	Equities	USD
3	Platinum	Commodities	USD	16	S&P/TSX 60	Equities	CAD
4	Silver	Commodities	USD	17	SMI	Equities	CHF
5	WTI Crude Oil	Commodities	USD	18	SPI 200	Equities	AUD
6	DAX	Equities	EUR	19	Australia 10Y Govt Bonds	Fixed Income	AUD
7	EURO STOXX 50	Equities	EUR	20	Canada 10Y Govt Bonds	Fixed Income	CAD
8	FTSE 100	Equities	GBP	21	Germany 10Y Govt Bonds	Fixed Income	EUR
9	Hang Seng	Equities	HKD	22	Germany 30Y Govt Bonds	Fixed Income	EUR
10	IBOVESPA	Equities	BRL	23	UK 10Y Govt Bonds	Fixed Income	GBP
11	MSCI Emerging Markets	Equities	USD	24	USA 10Y Govt Bonds	Fixed Income	USD
12	NASDAQ-100	Equities	USD	25	USA 30Y Govt Bonds	Fixed Income	USD
13	Nikkei 225	Equities	JPY				

Dataset starts in 1995.
Costs considered: 1 tick per transaction
Target daily volatility: 5%
Futures portfolio rebalanced once a month

Long-only portfolio

	Other Allocations						Network-based Allocations			
Metric	EW	NRP	MVP	MDP	ERC	HRP	IDCP$^{\rho}$	IECP$^{\rho}$	IECP^{l}	IECP$^{\rho,\tau,l}$
CAGR	3.19%	4.17%	4.35%	4.48%	4.62%	4.36%	4.35%	4.92%	4.56%	4.79%
Volatility	4.83%	4.90%	5.08%	5.06%	4.96%	4.98%	4.92%	4.95%	4.95%	4.95%
Skewness	-0.49	-0.52	-0.52	-0.44	-0.48	-0.41	-0.48	-0.49	-0.41	-0.45
Sharpe Ratio	0.66	0.84	0.85	0.87	0.92	0.86	0.87	0.98	0.91	0.96
Sortino Ratio	1.05	1.35	1.36	1.42	1.49	1.41	1.40	1.59	1.47	1.55
Calmar Ratio	0.16	0.25	0.35	0.39	0.33	0.51	0.26	0.44	0.33	0.38
CVaR(95%)	-10.6%	-9.0%	-5.8%	-8.0%	-7.7%	-4.1%	-8.6%	-5.8%	-6.8%	-6.0%
Max Drawdown	-20.6%	-17.1%	-12.8%	-11.7%	-14.4%	-8.7%	-17.2%	-11.3%	-14.2%	-12.8%
Transaction Costs (p.a.)	0.12%	0.16%	0.25%	0.29%	0.20%	0.23%	0.20%	0.23%	0.25%	0.22%

Long-only portfolio



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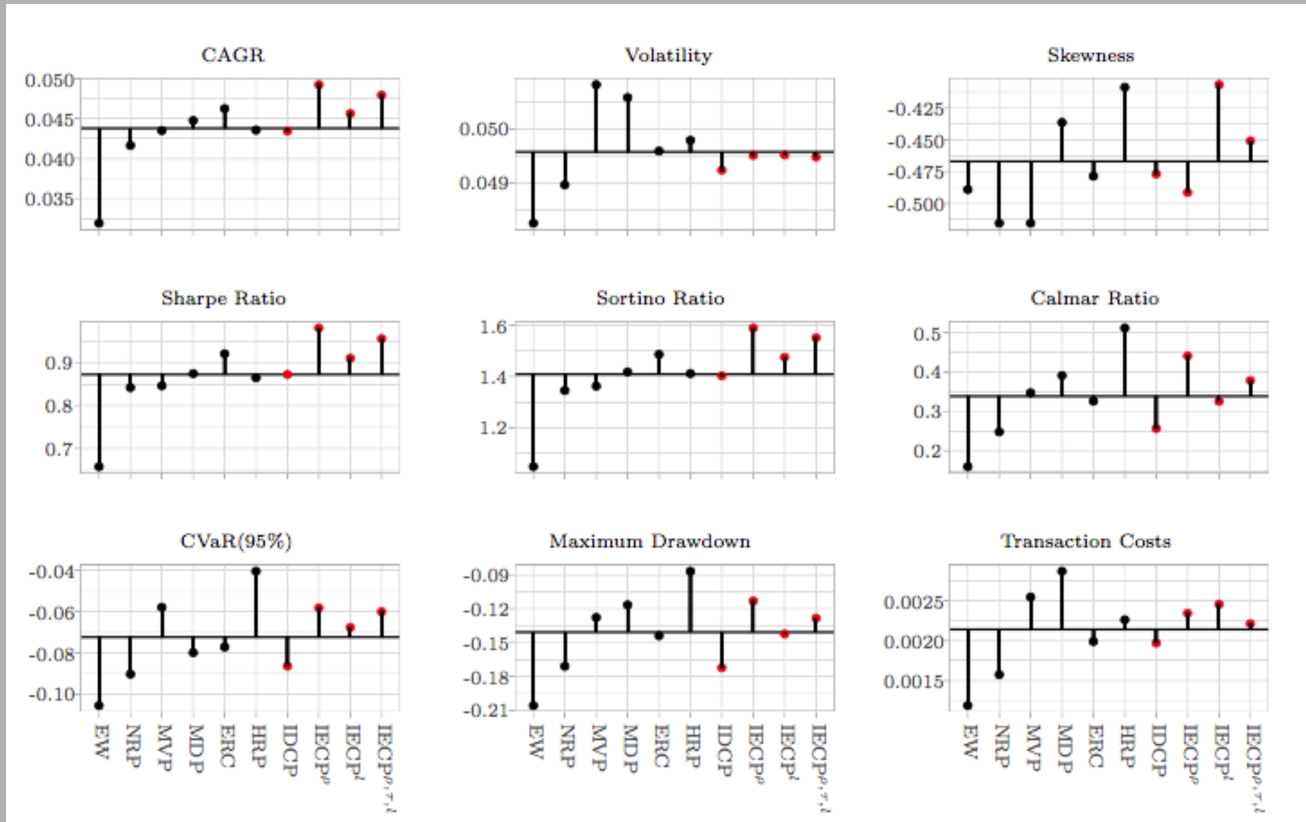

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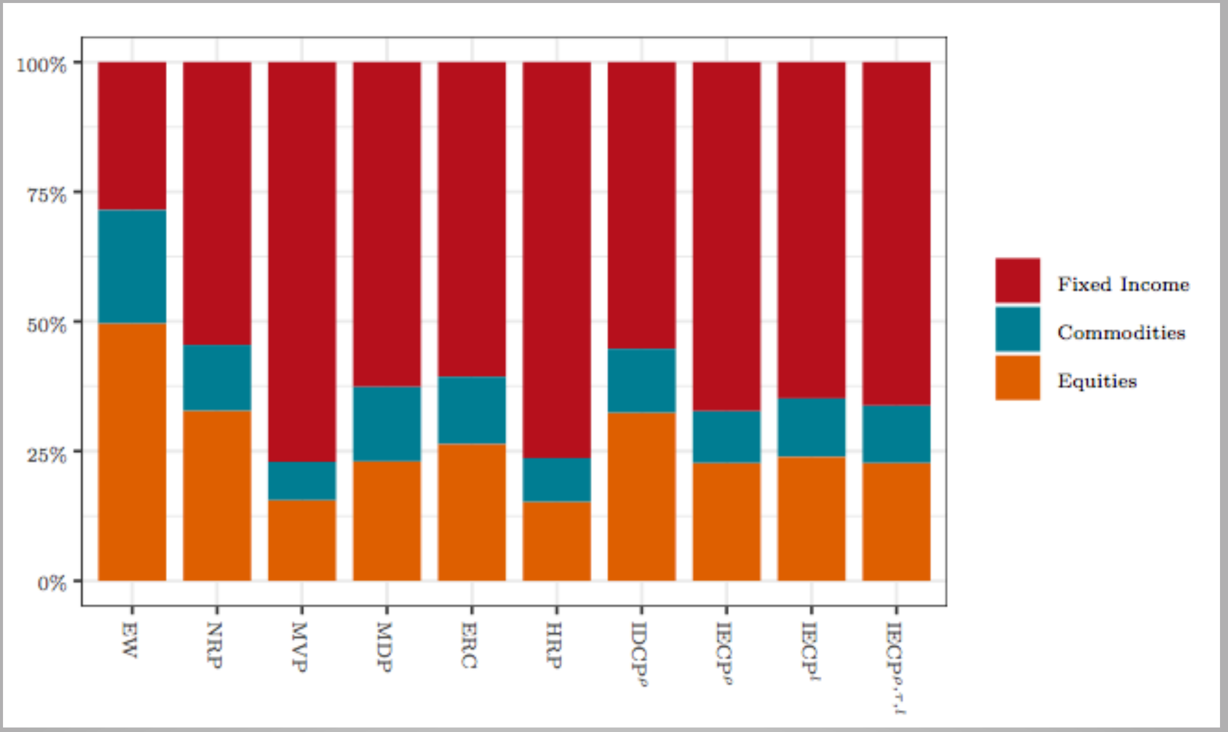

Long-only portfolio

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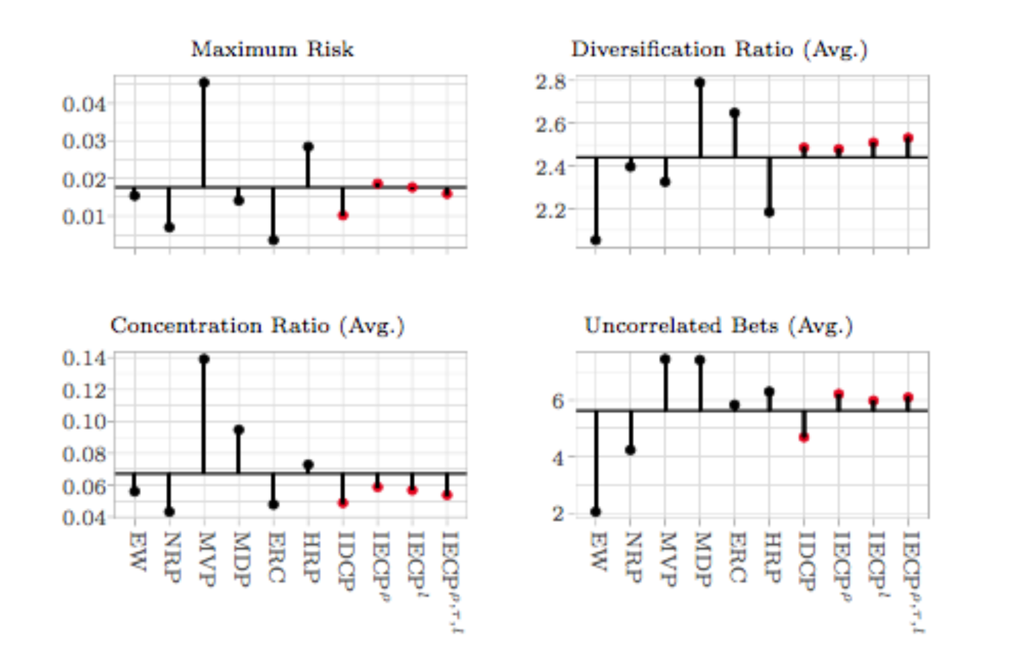
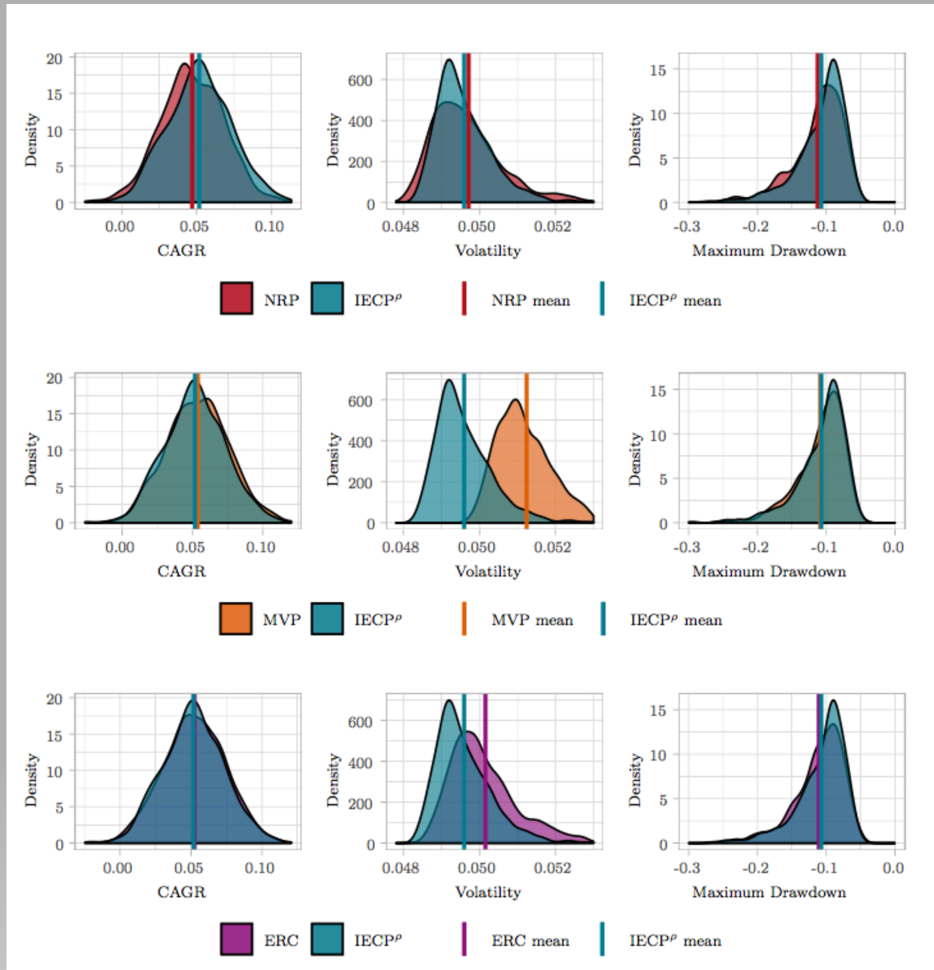
Maximum Risk (Avg.)	0.64%	0.36%	1.59%	0.71%	0.21%	0.94%	0.49%	0.75%	0.71%	0.61%
Diversification Ratio (Avg.)	2.05	2.4	2.33	2.79	2.65	2.18	2.49	2.48	2.51	2.53
Concentration Ratio (Avg.)	0.06	0.04	0.14	0.09	0.05	0.07	0.05	0.06	0.06	0.05
Uncorrelated Bets (Avg.)	2.06	4.24	7.45	7.42	5.83	6.3	4.69	6.21	5.98	6.1



$$DR(w) = \frac{\sum_i w_i \sigma_i}{\sqrt{w' \Sigma w}}$$

$$CR(w) = \frac{\sum_i (w_i \sigma_i)^2}{(\sum_i w_i \sigma_i)^2}$$

Long-only portfolio



Time-series portfolio

Jegadeesh and Titman (1993), Moskowitz, Ooi, and Pedersen (2012) and Asness, Moskowitz, and Pedersen (2013): momentum strategies may produce positive returns.

Strategy used:
4 different periods: 65, 125, 185 and 250 business days.
- Momentum is positive for at least 3 of these periods, signal = 1
- Momentum is negative for at least 3 of these periods, signal = -1
- Otherwise signal = 0

EW, NRP, HRP, network methods: invert the returns when signal = -1
MVP, MDP, ERC, constraint stemming from the signal:

$$w_i > 0, \quad \text{if } \text{signal}_i = 1$$
$$w_i < 0, \quad \text{if } \text{signal}_i = -1$$

Time-series portfolio

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2	Copper	Commodities	USD	20	NASDAQ-100	Equities	USD
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18	KOSPI 200	Equities	KRW				

Time-series portfolio

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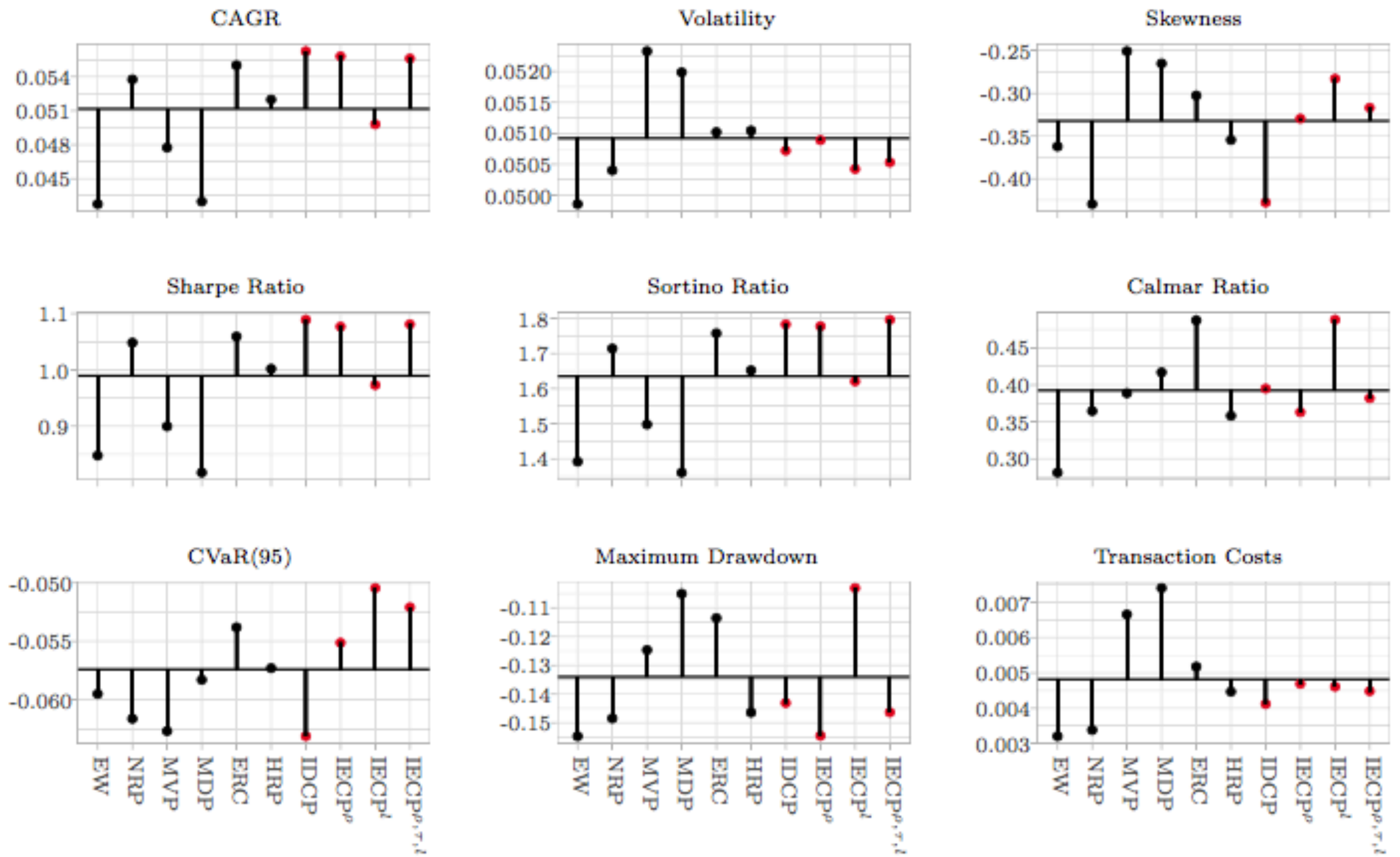
CAGR	4.28%	5.38%	4.77%	4.30%	5.50%	5.20%	5.62%	5.58%	4.98%	5.56%
Volatility	4.99%	5.04%	5.23%	5.20%	5.10%	5.10%	5.07%	5.09%	5.04%	5.05%
Skewness	-0.36	-0.43	-0.25	-0.27	-0.30	-0.35	-0.43	-0.33	-0.28	-0.32
Sharpe Ratio	0.85	1.05	0.90	0.82	1.06	1.00	1.09	1.08	0.97	1.08
Sortino Ratio	1.39	1.71	1.50	1.36	1.76	1.65	1.78	1.78	1.62	1.80
Calmar Ratio	0.28	0.36	0.39	0.42	0.49	0.36	0.39	0.36	0.49	0.38
CVaR(95%)	-5.95%	-6.16%	-6.27%	-5.83%	-5.38%	-5.73%	-6.31%	-5.51%	-5.04%	-5.21%
Max Drawdown	-15.5%	-14.8%	-12.5%	-10.5%	-11.4%	-14.6%	-14.3%	-15.5%	-10.3%	-14.6%
Transaction Costs (p.a.)	0.32%	0.34%	0.67%	0.74%	0.52%	0.45%	0.41%	0.47%	0.46%	0.45%

Time-series portfolio



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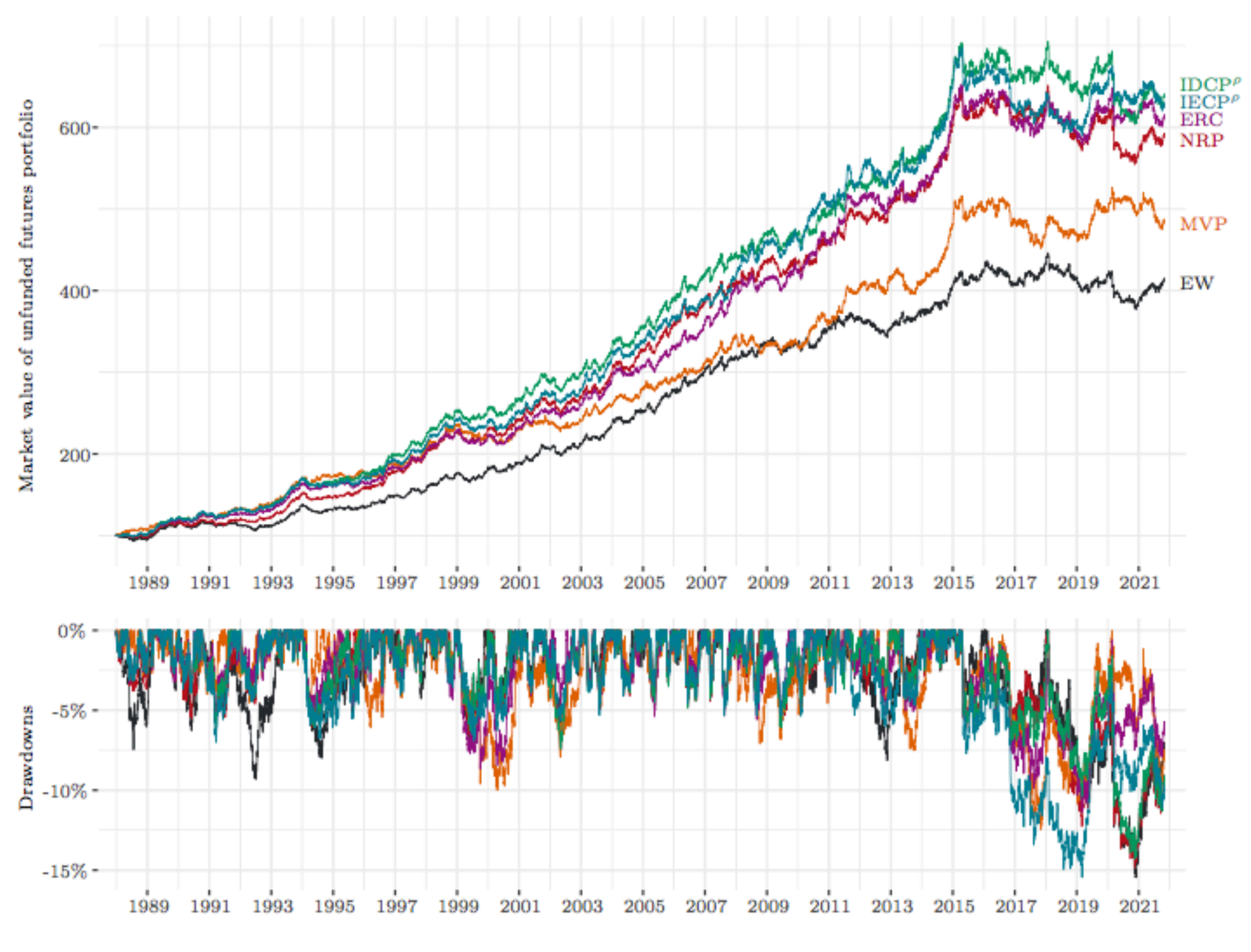

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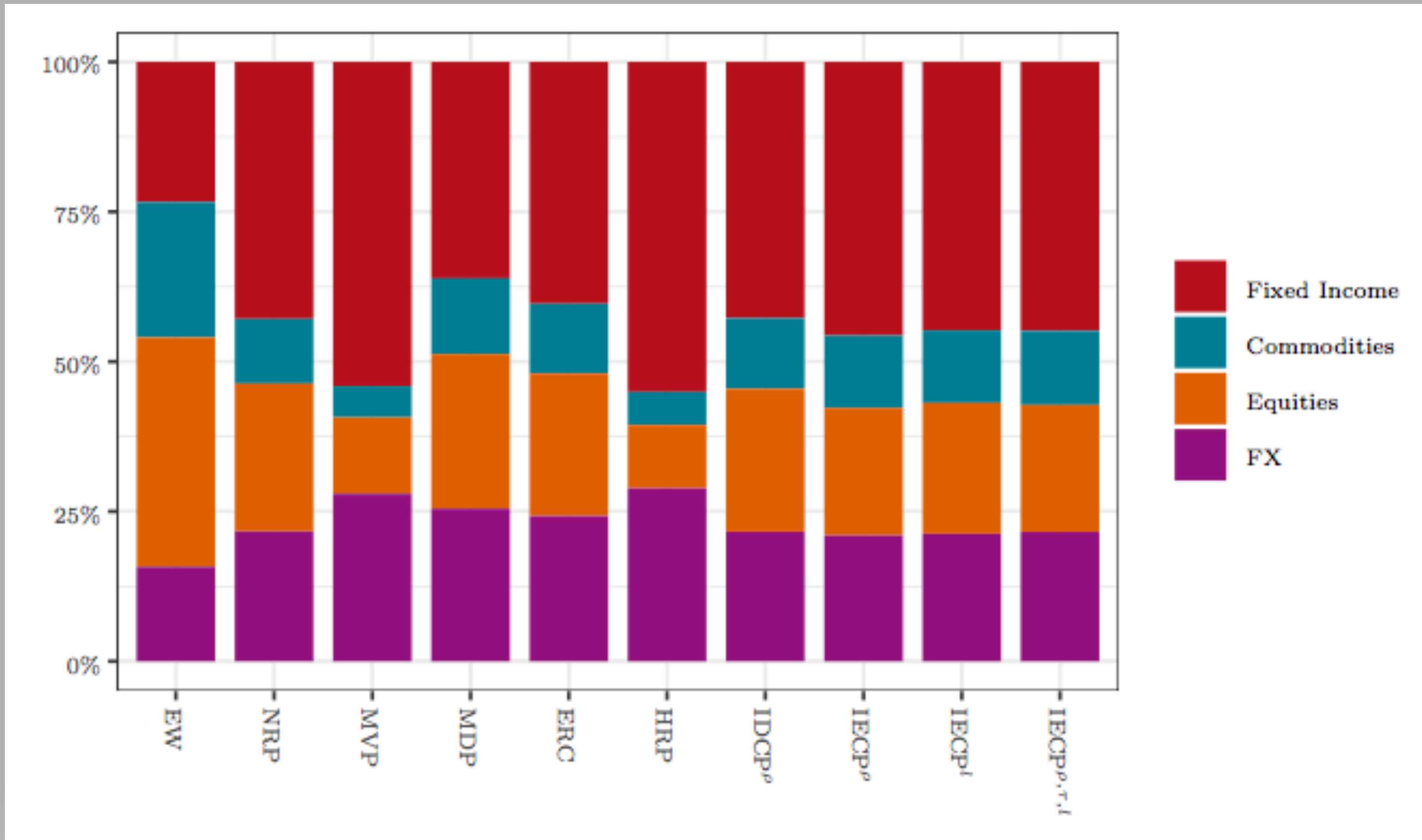

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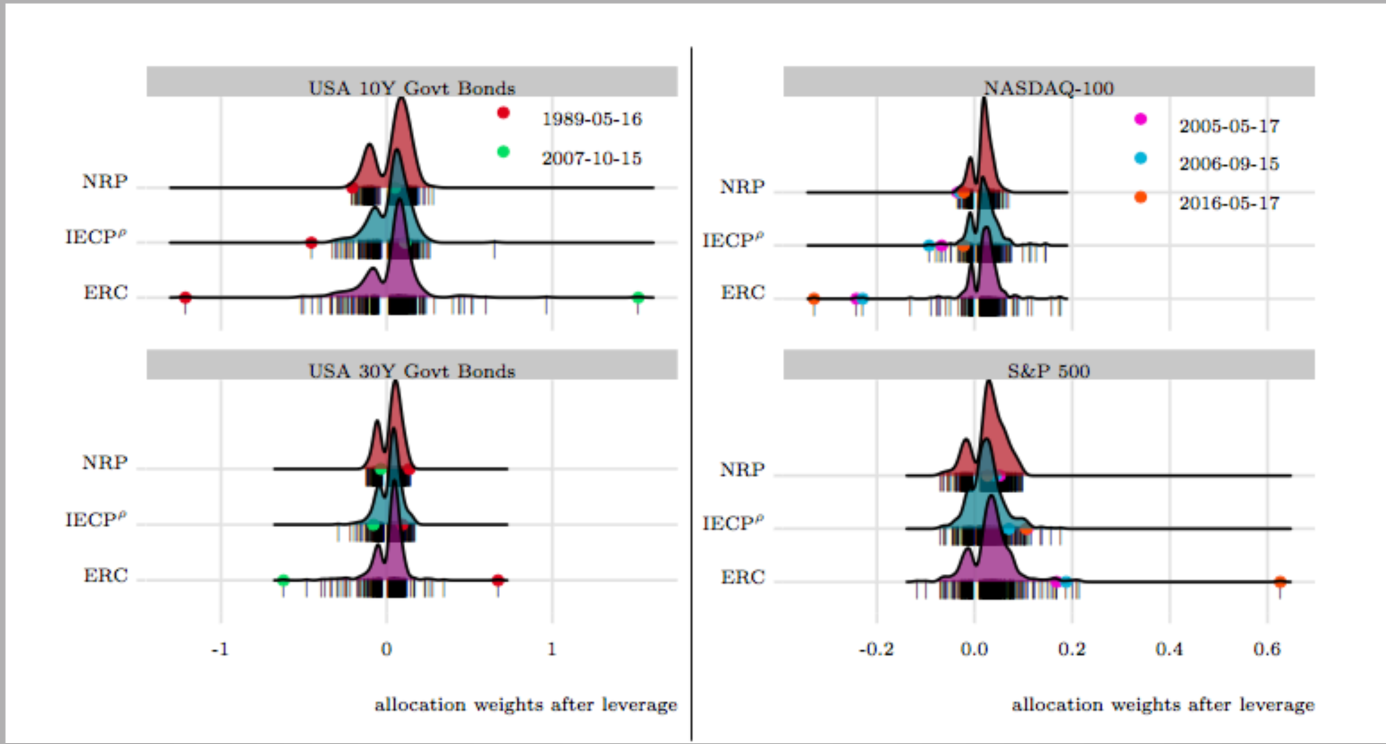

Problems when there is a long and short positions in very correlated assets, for optimizing algorithms in MVP, MDP and ERC.

Extension

- Other dependence measures (like partial correlations Kenett et al. (2010)).

- Different algorithms, like Maximally Filtered Clique Forest (Massara and Aste (2019)).

- Different centrality measures (Katz centrality, betweeness centrality, expected force).

-Coloured networks, where the weights are used in the estimation of the centrality measure.

Conclusion

- A filtered graph can uncover and retain the most important relationships within a portfolio.

- These sub-graphs seem to provide more stable structures than the complete graph (full correlation matrix), specially out-of-sample.

- The methods can be applied not only to long-only portfolios, but also long-short portfolios, where correlation-based allocation methods my run easily into extreme positions.

- Backtest suggest that proposed method have the potential to outperform competing traditional portfolio allocation techniques.